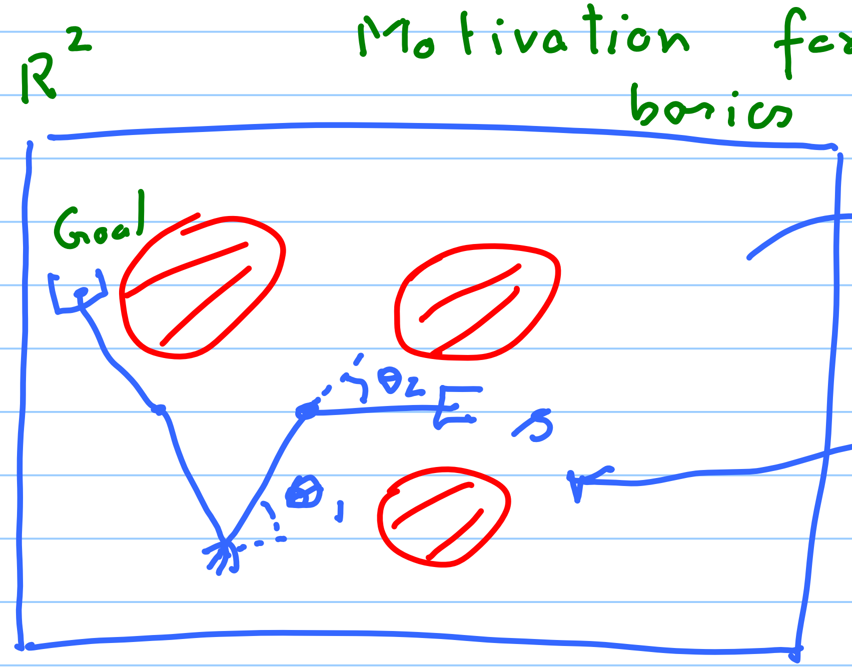
Note Title 1/12/2012

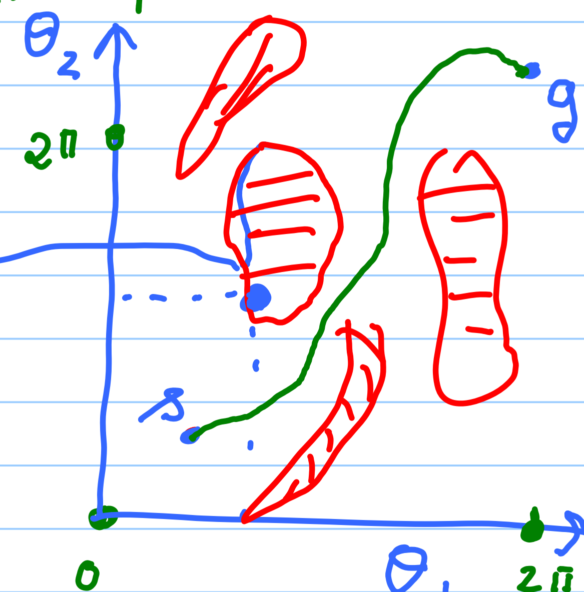
Lecture 3

Basic Motion Planning (MP) Problem

Motivation for why we cover the basics in next few lectures



Path Planning in Workspace



Configuration Space

Questions :

1) Nature of xformation
or mapping

2) structure of config space
("surface")

3) Does a path exist ?

1) Basic notions from math (distance, continuity, mappings)

2) " " " geometry

3) graph search techniques

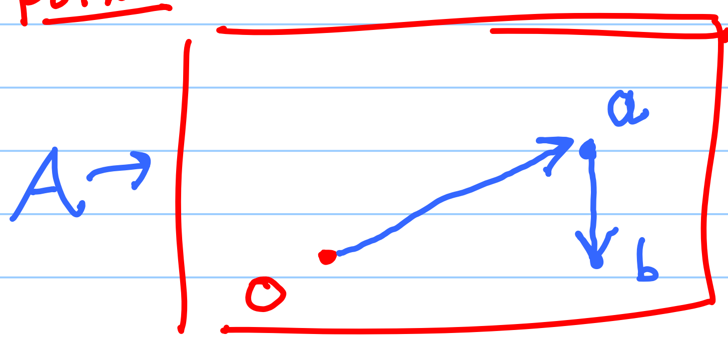
I) Basic notions from math:

1) vector space: assume known

2) Affine space :

"sets up a correspondence between "points" in a set and vectors in a vector space.

elements of A are points



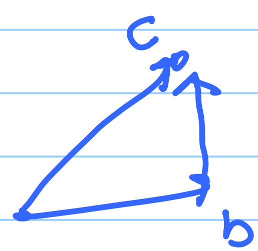
$\vec{Oa}$

Let E be a vector space. The affine space assoc with E is a set A :

1) every pair of pts $(a,b) \in A \times A$ determines a unique vector $\vec{ab} \in \vec{E}$

2) every pair $(a, \vec{x}) \in A \times \vec{E}$ def. a unique pt. b : $\vec{ab} = \vec{x}$

3) Law of paroll. of vect. add.

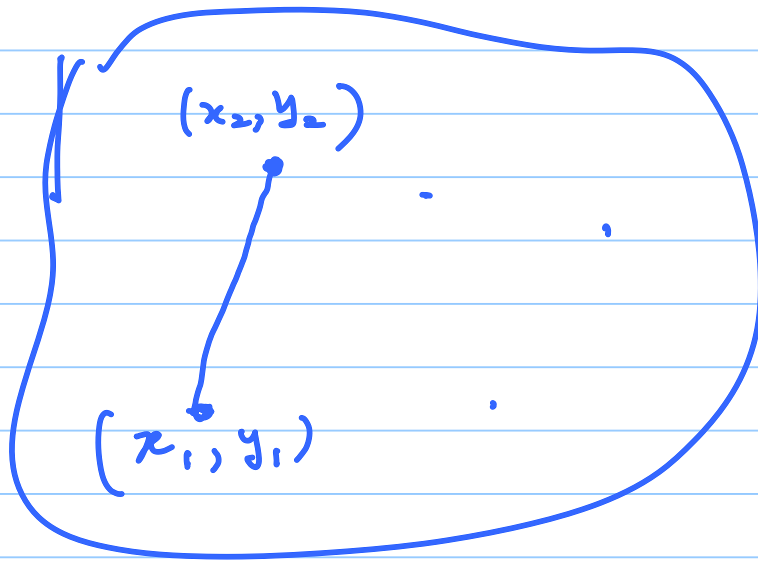


$$\vec{ac} = \vec{ab} + \vec{bc}$$

4) inverse :

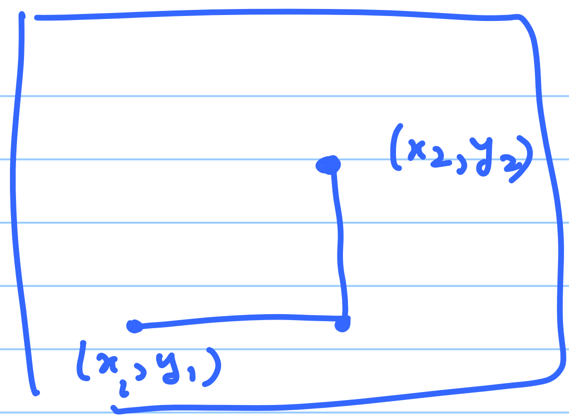
3) <u>Metric</u> or <u>Distance</u> over a set.

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} = d\left[(x_1, y_1), (x_2, y_2)\right]$$



$d : E \times E \to \mathbb{R}$ is a real valued funct. that satisfies :

Example: Manhattan or city block dist.



$$d\left[(x_1, y_1), (x_2, y_2)\right] = |x_1 - x_2| + |y_1 - y_2|$$

: prop. of $d(x, y)$ $x, y \in E$

1) +ve ness : $d(x, y) \geq 0$

2) non-degenerate : $d(x, y) = 0$ iff $x = y$

3) symmetric : $d(x, y) = d(y, x)$

4) triangular inequality:

$$d(x,y) + d(y,z) \geq d(x,z)$$

"shortest"

$E, d \longrightarrow$ metric space

Euclidean metric: $\sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$

$x = (x_1, x_2)$

$y = (y_1, y_2)$

L_p : $d_p(x,y) = \sqrt[p]{\sum_{i=1}^{m} |x_i - y_i|^p}$

$p = 2$ Euclidean

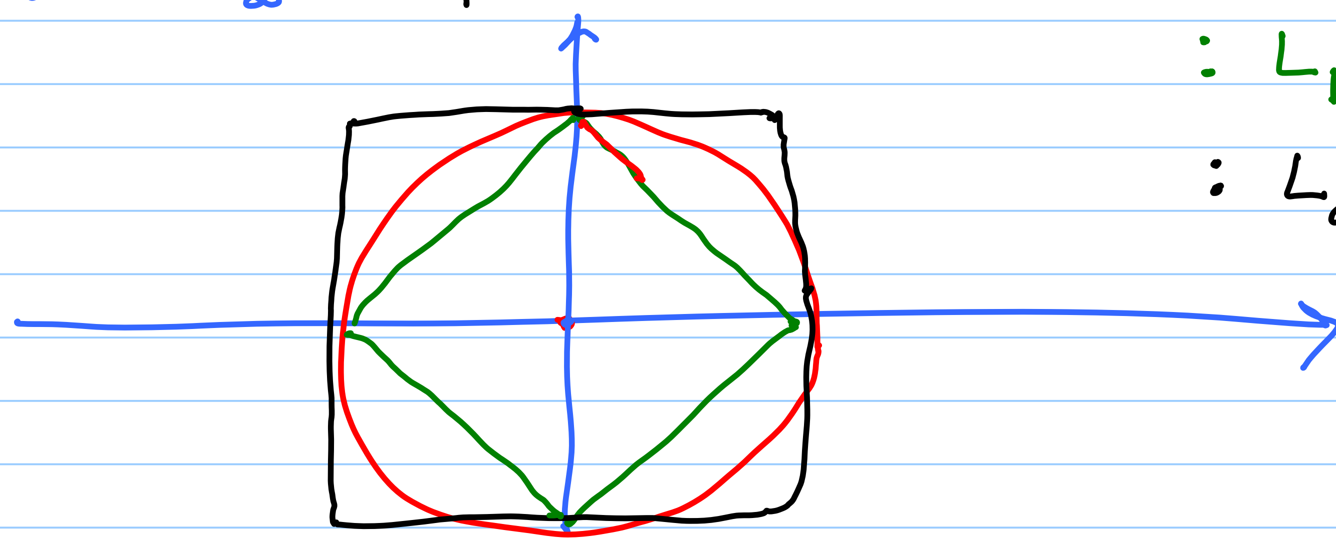
$p = 1$ City Block

$p = \infty$ $L_\infty \rightarrow \max_{i=1}^{m} |x_i - y_i|$

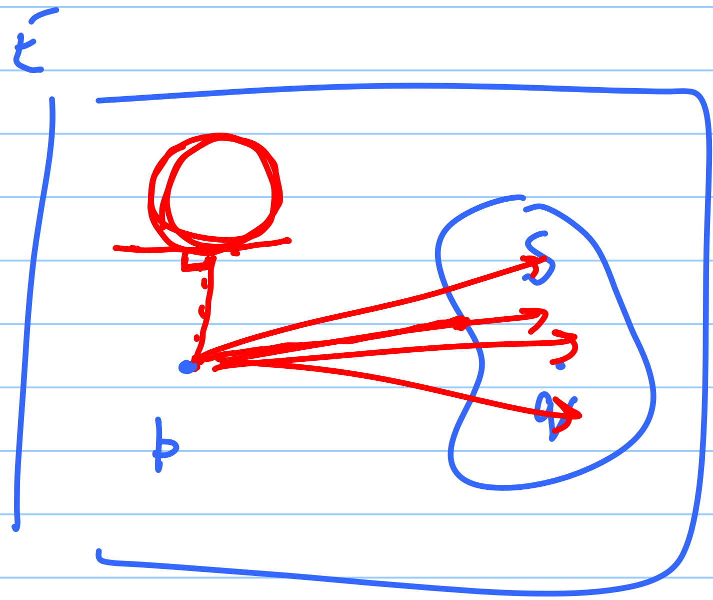
Unit circle : L_2

: L_1

: L_∞



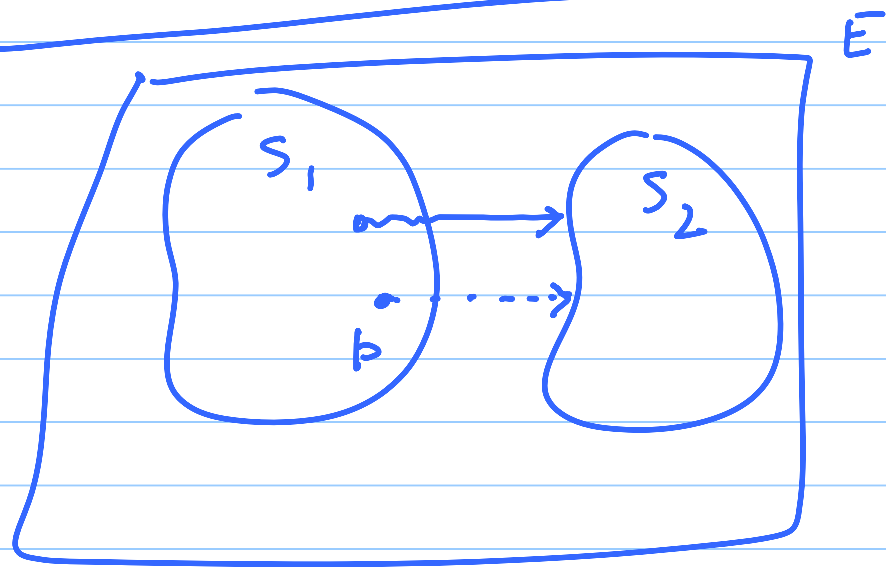
Distance bet. a point + a set:



$d(p, S)$:

$$\min_{q \in S} d(p,q)$$

dist. bet. two sets: $d(S_1, S_2)$: ?



Hausdorff distance: $\max_{p \in S_1} d(p, S_2)$

not symmetric: $d(S_1, S_2) \neq$

$d(s_2, s_1)$

inner product $\longrightarrow$ generalization of "angle"

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

$$\begin{matrix} \shortparallel & \shortparallel \\ 1 & 1 \end{matrix}$$

set E : $x, y \in E$ $\quad \langle x, y \rangle : E \times E \to \mathbb{R}$

such that

1) $\langle x, x \rangle \geqslant 0$

2) non-degenerat : $\langle x, x \rangle = 0$ iff $x = \underline{0}$

3) symm : $\langle x, y \rangle = \langle y, x \rangle$

4) bilinear:

$$\langle \lambda_1 x_1 + \lambda_2 x_2 , y \rangle$$

$$= \lambda_1 \langle x_1 , y \rangle + \lambda_2 \langle x_2 , y \rangle$$

space of real valued functions

$$[0,1] \to R$$

$$\langle f, g \rangle = \int_0^1 f(x)\, g(x)\, dx$$

"Length" $\longrightarrow$ Norm

$x \in E$, $\quad \|x\| : E \to \mathbb{R}$

such that

1) +ve : $\|x\| \geq 0$

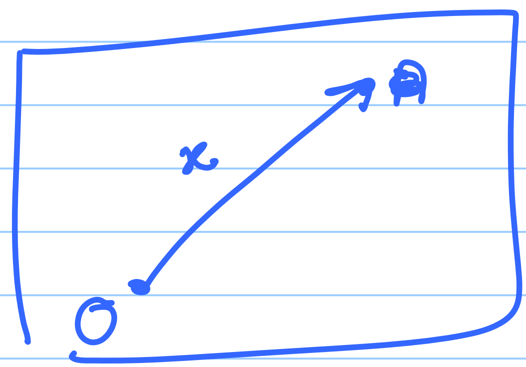
2) non-deg : $\|x\| = 0$ iff $x = \underline{0}$

3) $\|\lambda x\| = |\lambda| \, \|x\|$

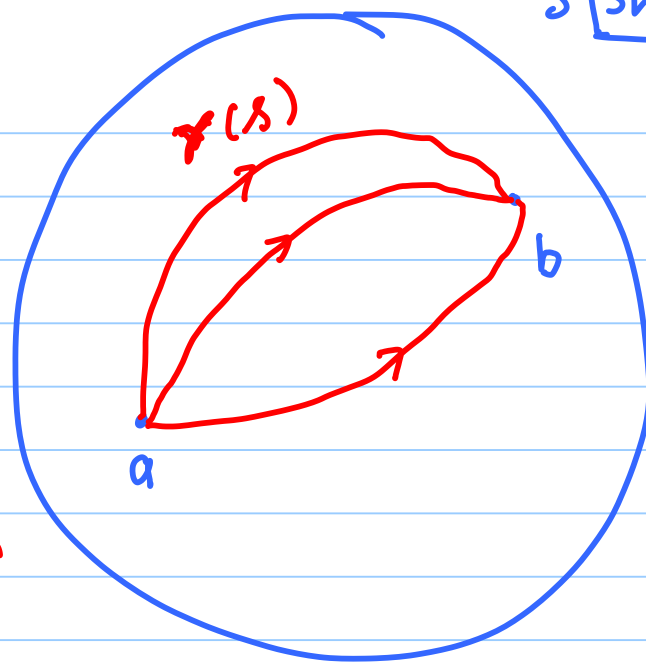
4) $\|x + y\| \leq \|x\| + \|y\|$

Inner product $\xrightarrow{\text{induce}}$ norm $\longrightarrow$ metric

$$\|x\| = \langle x, x \rangle$$



Sphere : geodesics "shortest length curves bet. two pts" on a surface



s : arc length

$$L(\gamma) = \int_0^1 \left\| \frac{d\gamma}{ds} \right\| \cdot ds$$

$$d(a,b) = \min_{\gamma} L(\gamma)$$

Elementary point set topology

assume that a metric space is given

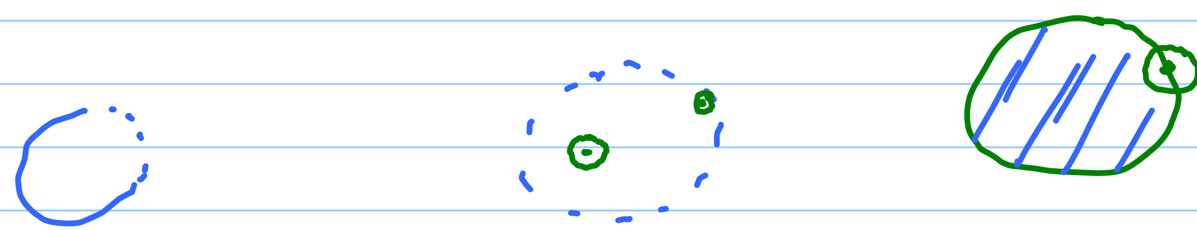
$d, \underline{S} \to$ metric space

$B_\varepsilon(p)$: open ε - ball centred at $p \in S$: $\{ q \in S : d(p,q) < \varepsilon \}$

1) open set: $A \subset S$

A is open if every pt. in A has an ε-ball (centred at the pt.) that belongs to A

$\varepsilon > 0$



2) limit point of A: A pt. p is a limit point of A if every ε-ball centred at the pt. contains at

least one (other than p) pt. $\in A$.

limit pt. of A does not have to belong to A

3) Closure of a set :

$$Cl(A) = A \cup \{\text{limit pts of } A\}$$

4) closed set : all limit pts of A $\in A$

5) <u>Boundary of a set:</u> ∂A

a pt. $\in \partial A$ if every ε-ball centred at the pt. has at least one pt. $\in A$ and at least one pt. $\notin A$.

6) interior of a set: $\text{Int}(A)$

a pt. $p \in \text{Int}(A)$ if

a ball centred at $p \subset A$

7) <u>exterior of A</u> : a pt. $p \in$

Ext(A) if $\exists$ a ball centred at

p such that $B_\varepsilon(p) \cap A = \phi$

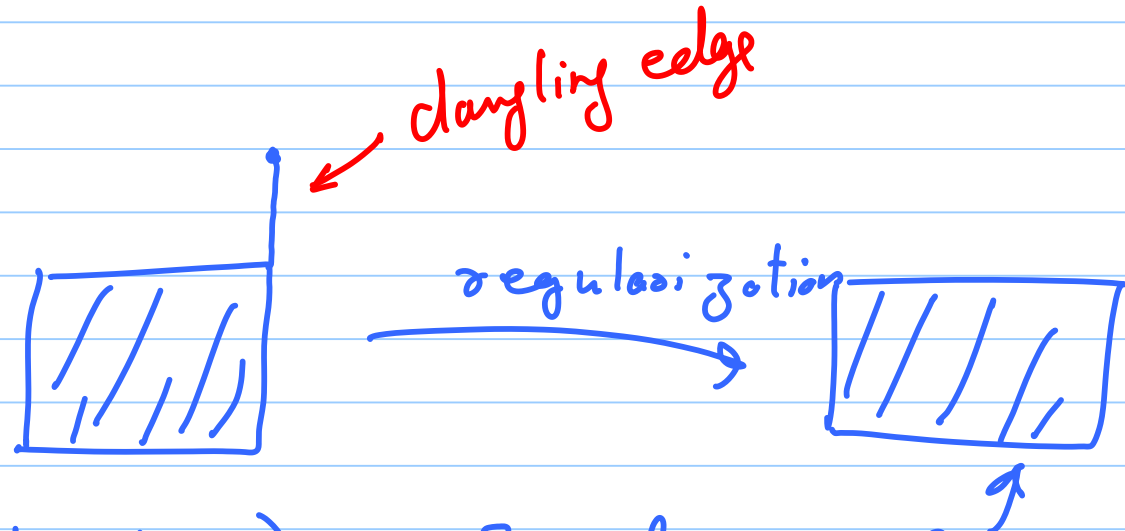
8) Bounded set : A is bounded

if it is contained in

some ball of finite radius

9) compact set: closed & bounded

10) A



$Cl(Int(A))$ = Regularized A